COMMUNICATIONS TO THE EDITOR

The Effect of Surfactants on the Flow Characteristics of Falling Liquid Films

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In a recent publication (*1*) some conclusions were drawn regarding the effect of surfactants on the entrance region flow of falling liquid films. There were two errors made in that publication which need to be corrected. The first error concerns the comparison of the experimental data of Lynn (*2*) with the approximate solution presented by Strobel and Whitaker. In using Lynn's data it was not realized that the Reynolds number was defined as

$$N_{Re} = \frac{4<u>h_\infty}{\nu} \tag{1}$$

where h_∞ is the uniform film thickness, $<u>$ is the average velocity, and ν is the kinematic viscosity. This Reynolds number is a factor of 4 larger than the traditional Reynolds number used to characterize this type of flow. When this is taken into account the agreement between Lynn's data and our theory is shown in Figure 1. In that figure U_s, N_{Re}, and X are defined by

$$U_s = u_{\text{surface}}/u_\infty \tag{2}$$

$$N_{Re} = u_\infty h_\infty/\nu \tag{3}$$

$$X = x/h_\infty N_{Re} \tag{4}$$

Here u_∞ is the surface velocity for the uniform flow and is given by $u_\infty = 3<u>/2$; thus the Reynolds number is based on the surface velocity and is therefore one and a half times larger than the traditional Reynolds number. Correction of our erroneous interpretation of Lynn's data has led to rather nice agreement between theory and experiment; however, our joy is rather short-lived.

In the analysis of the tangential surface stress the boundary condition in reference *1* was given as

$$\left(\frac{\partial U}{\partial Y}\right) = N_{El}\, N_{Re}^{-2/3} \left(\frac{\partial \Gamma}{\partial X}\right), \quad Y = 0 \tag{5}$$

when in fact the correct version is

$$\left(\frac{\partial U}{\partial Y}\right) = N_{El}\, N_{Re}^{-5/3} \left(\frac{\partial \Gamma}{\partial X}\right), \quad Y = 0 \tag{6}$$

This means that the surface elasticities used in our calculations were *too large* by a factor of N_{Re}, or by a factor of 100 for the calculations shown in Figure 10 of reference *1*. Our original conclusion was that the entrance length for a 3×10^{-3} molar hexanoic acid solution might be on the order of 50 cm. for $N_{Re} = 100$; however, the conclusion based on the correct boundary condition given by Equation (6) is that the entrance length for these same conditions is approximately 1 cm. This has been confirmed by recent experiments (*3*).

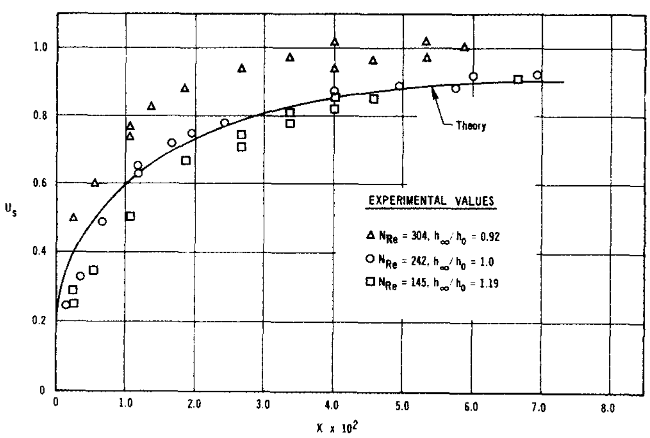

Fig. 1. Comparison between the experimental data of Lynn and the one-dimensional theory of Strobel and Whitaker.

NOTATION

- g = gravitational constant, cm./sec.^{2}
- h_∞ = film depth for uniform flow, cm.
- N_{El} = $(-\partial\sigma/\partial\Gamma)(2/g\,\nu^4\rho^3)^{1/3}$, dimensionless elasticity number
- U = v_x/u_∞, dimensionless x component of the velocity vector
- Y = y/h_∞, dimensionless distance orthogonal to the main flow
- ρ = fluid density, g./cu.cm.
- Γ = γ/γ_0, dimensionless surface mass density
- σ = surface tension, dyne/cm.

LITERATURE CITED

1. Strobel, W. J., and Stephen Whitaker, *AIChE J.*, **15**, 527 (1969).
2. Lynn, S., *ibid.*, **6**, 703 (1960).
3. Cerro, R. L., Ph.D. thesis, Univ. California, Davis (1970)